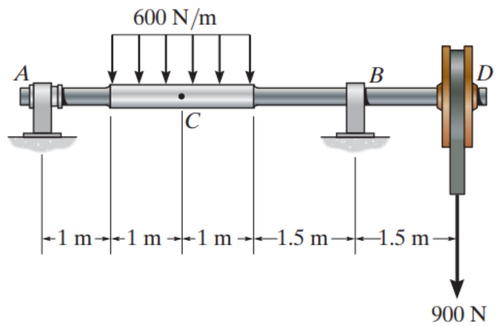
Problem 1-4

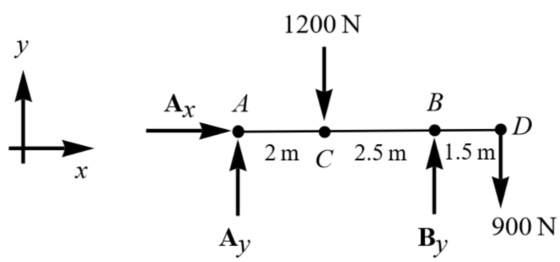
The shaft is supported by a smooth thrust bearing at A and a smooth journal bearing at B. Determine the resultant internal loadings acting on the cross section at C.



Prob. 1–4

Solution

The thrust bearing at A prevents horizontal and vertical motion, and the journal bearing at B prevents vertical motion. As a result, there are three unknown forces in the free-body diagram of the shaft shown below.



The distributed force is treated as a single resultant force applied through the centroid of the rectangle to point C with a magnitude given by the area of the rectangle,

$$A = \left(600 \ \frac{\text{N}}{\text{m}}\right)(2 \text{ m}) = 1200 \text{ N}.$$

Use the equations of equilibrium to determine the reaction forces, taking the sum of the moments about point A.

$$\sum F_x = A_x = 0$$

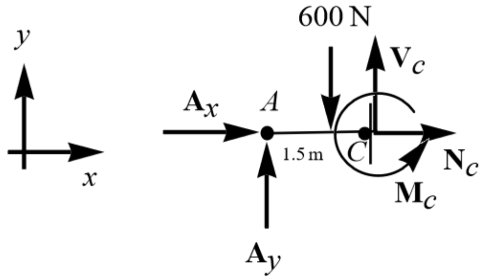
$$\sum F_y = A_y - 1200 + B_y - 900 = 0$$

$$\circlearrowleft^{+} \sum M_A = -(1200 \text{ N})(2 \text{ m}) + B_y(4.5 \text{ m}) - (900 \text{ N})(6 \text{ m}) = 0$$

Solving this system of equations yields

$$A_x = 0 \quad \text{and} \quad A_y = \frac{1100}{3} \approx 367 \text{ N} \quad \text{and} \quad B_y = \frac{5200}{3} \approx 1733 \text{ N}.$$

Now that the reactions at A and B are known, the resultant internal loadings acting on the cross section at C can be determined using the method of sections. Since it's simpler, choose the section of the shaft from B to C.



This 600-N force is the resultant force of the part of the distributed force from A to C. The force's magnitude is the area,

$$\left(600\ \frac{\text{N}}{\text{m}}\right)(1\text{ m}) = 600\text{ N},$$

and the force passes through the centroid, which is 1.5 m from A. Use the equations of equilibrium to find the internal loadings at C.

$$\sum F_x = A_x + N_c = 0$$

$$\sum F_y = A_y - 600 + V_c = 0$$

$$\circlearrowleft^{+} \sum M_C = M_c + (600\text{ N})(0.5\text{ m}) - A_y(2\text{ m}) = 0$$

Solving this system of equations yields

$$N_c = -A_x = 0$$

$$V_c = 600 - A_y = \frac{700}{3} \approx 233\text{ N}$$

$$M_c = 2(-150 + A_y) = \frac{1300}{3} \approx 433\text{ N}\cdot\text{m}.$$